Sets of situations, topics, and question relevance

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Abstract. In this paper we provide formal tools for analyses of question processing involved in solutions to a specific class of abductive problems. We model this processing in terms of relations of sifting and funneling, for definitions of which we employ logic of questions, situational semantics, and topic relevance. Our empirical evidence consists of logs of gameplays of the game *Mind Maze* by Igrology.

Keywords. logic of questions, topic relevance, situational semantics

In this paper we provide formal tools for analyses of question processing involved in solutions to a specific class of abductive problems. We model this processing in terms of relations of sifting and funneling. Definitions of these relations employ logic of questions, situational semantics, and topic relevance. In the section 1 we briefly outline the source of our data. In the section 2 we characterize formal tools used in analysis of the data. On this basis, in the sections 3 and 4 the concepts of topic and question relevance, and of an admissible answer are introduced, respectively. In the sections 5 and 6 we give analyses of exemplary *20 questions* and *Mind Maze* gameplays in these terms. We conclude the paper by pointing at some directions for further research.

1. The source empirical data

We devised materials for this research on the basis of *Mind Maze* tasks. This is a game by Igrology in which, according to the manual, a gamemaster "describes a strange story and the players must determine why and how the story happened". Solution of each of the tasks is dependent on discovering key pieces of information (which are known to the gamemaster only) by asking auxiliary questions. Thus the task of the player is to process a sequence of questions, posed on the basis of a story's content and subsequent answers of the gamemaster. The players may collaborate in order to reach the solution. We modified original rules of the game, in order to allow for more cooperative behaviour of a gamemaster as well as to smoothen the process of data gathering. In particular, as in the original version, we allow for only yes-no questions to be asked, but with addition of two admissible answers: "not important" and "it is not known". The interested reader will find the details on the setup of this research in [1]. Data obtained from *Mind Maze*

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gameplays form one of the three subcorpora of the Erotetic Reasoning Corpus [2]. *Mind Maze* subcorpus currently consists of 38 annotated gameplays (69.434 words), which lasted from 5 to 38 minutes. In order to clarify intuitions underlying our formal tools, before turning to a *Mind Maze* task (section 6), we introduce a toy example based on a *20 questions* gameplay (section 5).

2. The tools: questions, situations, and topics

As we demonstrated elsewhere [1], solutions to all the *Mind Maze* problems consisted of two general phases. They correspond to Stenning and van Lambalgen's [7] distinction of reasoning to vs reasoning from an interpretation. In the first phase the subjects established interpretation of a problem. As most of the subjects were not very explicit about underlying reasoning processes, first part of our model is based on limited amount of data and offers a rational reconstruction of this phase rather than its full-fledged descriptive model. We employ here elements of Gabbay and Woods' [4] formal schema of abductive reasoning and Kubiński's [5] logical theory of numerical questions. In the second phase, which consisted of the actual dialogues with the experimenter (gamemaster), the subjects' information processing can be adequately modelled by means of some extensions of the situational semantics [6], incorporating the concepts of topic and question relevance. In our preliminary report [1] we employed to this end Inferential Erotetic Logic [8]. However, it turned out that our present formalism is better suited for these purposes.

2.1. *Setting the cognitive goal: a formal schema of abduction*

In *Mind Maze* the players face an abductive problem of making sense of a puzzling fact [3] given in a story and expressed in the initial question. The task is to find pieces of information forming the explanation (as there is the correct solution to each problem in the game) of this puzzling fact. The players are supposed to rely on their general knowledge as well as on their abilities to reason with questions as premises and conclusions.

In order to account for this abductive nature of the tasks we employ some elements of Gabbay and Woods [4, p. 47] formal schema of abductive reasoning. The authors use the symbol ! to represent the fact that a certain piece of information (represented by T) is a cognitive target of a reasoning subject. Interpretation of this symbol, as well as of some others included in the schema, is not univocal: they may be sentences, theories or rules, or anything forming an abductive hypothesis in a certain context. We shall use the T symbol as a sentential metavariable. Thus in our analysis $T!$ will stand for propositionally construed cognitive target of a subject.

2.2. *Interpreting the problem: logic of questions*

In representing questions we adopt some essential elements of Kubiński's [5] analysis of simple numerical questions and also of propositional questions. We shall use simple numerical questions in order to account for the reasoning to an interpretation phase of a problem solution [7, p. 19–25], in which the subjects need to establish what key pieces of information they need to gather in order to obtain the solution. Propositional questions we shall be using are exclusively simple yes-no questions; a question with A and $\neg A$ as the only direct answers will be represented by $?A$ (which is a slight simplification of

Table 1. Examples of simple and compound numerical question [8, p. 54]

	Question	Possible reading
1.	$k \leq x_i Px_i$	for which [at least k] x_i, Px_i?
2.	$k < x_i Px_i$	for which [more than k] x_i, Px_i?
3.	$kx_i Px_i$	for which [exactly k] x_i, Px_i?
4.	$Cx_i Px_i$	for which [all] x_i, Px_i?
5.	$(k \leq)x_i Px_i$	which are all [at least k] x_i such that Px_i?
6.	$(k <)x_i Px_i$	which are all [more than k] x_i such that Px_i?
7.	$(k)x_i Px_i$	which are all [exactly k] x_i such that Px_i?
8.	$[1]1x_i, 1x_j Px_i x_j$	Which one x_i and one x_j are such that $Px_i x_j$?
9.	$[1]3x_i, 1x_j Px_i x_j$	Which three x_i and one x_j are such that $Px_i x_j$?
10.	$[2]3x_i, 3x_j Px_i x_j$	Which three x_i and three x_j are such that $Px_i x_j$ and each of the x_i is associated with exactly two x_j?

the original Kubiński's formalism). However, we need to extend the original Kubiński's concept in order to accomodate into our framework not only direct answers to such questions but admissible ones as well. The interested reader may find a concise summary of Kubiński's formalism in Wiśniewski [8, p. 52–62].

On Kubiński's account a *simple numerical question* is an expression of the form $Ox_i Px_i$, where Px_i (a *desideratum* of a question, as we shall call it after [9]) is a sentential function with x_i as the only free variable and Ox_i is a simple numerical operator containing x_i as the only variable. Some examples of questions so construed are given in the table 1. Items 1.–7. are examples of simple numerical questions, whereas 8.–10. are examples of compound numerical questions.

We shall follow the general idea of this formalism. However, it needs to be modified in order to model the cognitive tasks the subjects were solving in *Mind Maze*.

Firstly, on Kubiński's account numerical questions ask for lists of objects exhibiting certain properties. Thus direct answers (defined syntactically) to numerical questions are build by means of first-order sentences in which free variables indicated in a question are either replaced by closed terms or quantified. In *Mind Maze* questions concern pieces of information needed to attain the subject's target and expressed in sentences. As a result, instead of first-order representation, we will be employing propositional one.

Secondly, there is often more than one piece of information needed in order to attain the subject's target, and those pieces are not easily combined in a way analogous to the one offered by compund numerical questions. Thus we allow for multiple question-forming operators to be put in front of desideratum of a question.

We will be using questions analogous to no. 7. in the table 1., of the form: $(k)\alpha\beta$, where both α and β are formulas of some assumed language and α occurs in β; we are not using the notion of subformula beacuse we want to allow the 'assumed language' to be not only some object-level formal language (the language of Classical Propositional Calculus, CPC, in particular) but a metalanguage as well (incorporating elements of situational semantics). An example of such a question is an expression of the form: $(1)H_1\ X \models H_1$, which can be read as: 'Which is the only formula H_1 entailed by the set X?'. Arguably, the resulting way of representing questions does not meet the standards of the definition of a formalized language. Nevertehless, at the present stage of development of these instruments we consider their flexibility to be the main virtue to be pursued.

2.3. Information processing: situational semantics

In making sense of the concept of situation we shall follow Keith Devlin's claim that "situations are just that: situations" [10, p. 70], considering it as a primitive concept. We will employ Wiśniewski's [6] situational semantics, thus sharing his intuitions concerning the notion, of which the basic is that each atomic sentence refers to a set of situations: "If the relevant set is non-empty, then the set comprises all these situations in which (the claim made by) the atomic sentence holds" [6, p. 33]. Notice, that this has nothing to do with truth, as yet. However, "the relevant sets of situations are neither supposed to be non-empty nor have to be singleton sets" [6, p. 34].

A situational model of the CPC language is defined as follows [6, p. 39] ($Form_{CPC}$ stands for the set consisting of all and only CPC formulas).

Definition 1 *A* situational model *of the CPC language is an ordered pair* $\mathbb{M} = \langle U, v \rangle$*, where U is a non-empty set (the universe of* $\mathbb{M}$*) and v is a function* $Form_{CPC} \mapsto 2^U$ *such that:*

1. *for each propositional variable* p_i, $v(p_i) \subset U$
2. *for each* $A, B \in Form_{CPC}$*:*
   (a) $v(\neg A) = U - v(A)$,
   (b) $v(A \wedge B) = v(A) \cap v(B)$,
   (c) $v(A \vee B) = v(A) \cup v(B)$,
   (d) $v(A \rightarrow B) = v(\neg A) \cup v(B)$,
   (e) $v(A \leftrightarrow B) = (v(\neg A) \cup v(B)) \cap (v(\neg B) \cup v(A))$.

Definition 2 *A formula A is a* situational tautology *iff for every situational model* $\mathbb{M}$*:* $v(A) = U$.

Notice, that it is possible, that one atomic sentence subsumes another one, as in:

- p: Snoopy is an animal,
- q: Snoopy is a dog.

In this case $v(q) \subset v(p)$ (for a certain, rather commonsense model $\mathbb{M}$). This relation of subsumption may be interpreted as a kind of non-logical entailment. However, it is not tantamount to the intuitive natural-language concept of entailment, as, if A is a situational tautology, for any non-tautological formula B, $v(B) \subset v(A)$.

2.4. Topics

Interpreting the concept of topics in terms of situational semantics we shall follow some general lines proposed by Van Kuppevelt, according to whom "[t]he term topic (...) refer[s] to a topic notion which concerns the 'aboutness' of (sets of) utterances" [11, p. 111]. As the author claims:

> The notion presupposes that a discourse unit U – a sentence or a larger part of a discourse – has the property of being, in some sense, directed at a selected set of discourse entities (a set of persons, objects, places, times, reasons, consequences, actions, events or some other set), and not diffusely at all discourse entities that are introduced or implied by U. This selected set of entities in focus of attention is what U is about and is called the topic of U. [11, p. 112]

3. Relevance

3.1. Topic relevance

We shall be considering topics w.r.t. (or in) some situational model. On this account, topic Ω may be interpreted as a subset of the model's universe ($\Omega \subseteq U$). One distinguished class of topics, with which we will not be dealing here, are tautological topics, that is, topics which cover all the model's universe (Ω is a tautological topic iff $\Omega = U$).

Our notion of relevance is relative to a situational model and a topic in it.

Definition 3 *Let* $\mathbb{M} = \langle U, v \rangle$ *be a situational model and* $\Omega \subseteq U$ *be a topic in* $\mathbb{M}$. *A* situational relevance model *of* Ω *w.r.t.* $\mathbb{M}$ *is an ordered triple* $\mathbb{N} = \langle \Omega, w, \mathbb{M} \rangle$, *where w:* $Form_{CPC} \nrightarrow 2^{\Omega}$ *is such that:* (*) *if* $w(A) \subseteq \Omega$, *then* $w(A) = v(A)$.

Further on we shall refer to such models simply as to relevance models.

Definition 4 *A topic* Ω' *is* relevant *to a topic* Ω *w.r.t.* $\mathbb{N}$ *iff* $\Omega' \subseteq \Omega$.

Thus a formula A is relevant to a topic Ω w.r.t. $\mathbb{N}$ iff $w(A) \subseteq \Omega$. The condition imposed on w in definition 3 may be expressed as follows: (*') If A is relevant w.r.t. $\mathbb{N}$, then $w(A) = v(A)$.

Definition 5 *A topic* Ω *subsumes a topic* Ω' *w.r.t.* $\mathbb{N}$ *iff* $\Omega' \subseteq \Omega$.

As a result, Ω' is relevant to Ω iff Ω subsumes Ω' (w.r.t. $\mathbb{N}$). Both of these relations hold between two topics simultaneously iff the topics are identical.

In our analyses we found useful also a slightly weaker notion of relevance, which is given in def. 6. However, we shall not be using this weaker notion in the present paper.

Definition 6 *A topic* Ω' *is* somewhat relevant *to a topic* Ω *w.r.t.* $\mathbb{N}$ *iff* $\Omega' \cap \Omega \neq \emptyset$.

3.2. Question relevance

The idea underlying the notion of question relevance is that for a question to be relevant w.r.t. certain relevance model, at least one of its direct answers must be relevant w.r.t. that model.

Definition 7 *Let Q be a question and let* $\mathbf{d}Q = \{A_1, \ldots, A_n\}$ *be the set of all the direct answers to Q. Q is* relevant *w.r.t.* $\mathbb{N}$ *iff there exists* A_i ($1 \leq i \leq n$) *which is relevant w.r.t.* $\mathbb{N}$, *that is, such that* $w(A_i) \subseteq \Omega$.

Thus Q is not relevant w.r.t. $\mathbb{N}$ iff none of its direct answers is relevant w.r.t. $\mathbb{N}$.

Two relations by means of which we shall be modeling solutions to *Mind Maze* tasks, sifting and funneling, are in fact special cases of questions relevance. As only simple yes-no questions are allowed by the rules of the game we shall define these relations for such type of questions only.

Definition 8 $?A_1$, ..., $?A_n$ *are* sifting *questions w.r.t. a topic* Ω *of a certain relevance model* $\mathbb{N}$ *iff for every i and j* ($1 \leq i, j \leq n$)*:* $v(A_i)$ *is non-empty, and* $v(A_1) \subset \Omega$, ..., $v(A_n) \subset \Omega$, *and if* $A_i \neq A_j$, *then* $v(A_i) \cap v(A_j) = \emptyset$.

Thus $?A_1$, ..., $?A_n$ are sifting w.r.t. a topic Ω of a certain relevance model $\mathbb{N}$ iff sets of situations assigned to affirmative answers to these questions are pairwise disjoint and all are subsets of Ω; in other words, the sets $v(A_1)$, ..., $v(A_n)$ are partitioning Ω, albeit these partitioning need not to be exhaustive.

Definition 9 *A question* $?A$ funnels *a topic* Ω *of a certain relevance model* $\mathbb{N}$ *iff* $v(A) \subset \Omega$*, and both are non-empty. A question* $?A_1$ *funnels a question* $?A_2$ *w.r.t. a certain relevance model* $\mathbb{N}$ *iff* $v(A_1) \subset v(A_2)$*, and both are non-empty.*

Thus a question $?A$ funnels a topic Ω w.r.t. a certain relevance model $\mathbb{N}$ iff an affirmative answer to $?A$ narrows down Ω, and analogously in the case of funneling holding between questions. This relation is similar to the relation of cognitive usefulness, which holds between implied and implying questions in the case of erotetic implication [12, p. 72].

Examples of both of these relations are provided in the sections 5 and 6.

3.3. Truth values

In situational semantics it is quite natural to construe truth values in terms of partitions of a universe of situations: Partition of a universe U is an ordered pair $\mathbb{P} = \langle T_{\mathbb{P}}, F_{\mathbb{P}} \rangle$, such that:

1. $T_{\mathbb{P}} \cap F_{\mathbb{P}} = \emptyset$
2. $T_{\mathbb{P}} \cup F_{\mathbb{P}} = U$

Intuitively: $T_{\mathbb{P}}$ is the set of situations that hold and $F_{\mathbb{P}}$ is the set of situations that do not. Notice, that on this account both $T_{\mathbb{P}}$ and $T_{\mathbb{F}}$ are topics in U; this nicely coincides with Frege's [13] ideas on reference of sentences being truth values. Certainly, a suitably defined standard partition is needed in order to align assignment of truth-values to the concept of a situational model. The choice of the meaning of 'standard' depends on the choice of the underlying logic. What we found useful is the following definition of truth value of a formula A in a partition $\mathbb{P}$ of U, which nicely translates into Kleene's weak three-valued logic (**T** stands for truth, **F** for falsehood and **N** for the third value):

1. $V(A, \mathbb{P}) = \mathbf{T}$ iff $v(A) \neq \emptyset$ and $v(A) \subseteq T_{\mathbb{P}}$
2. $V(A, \mathbb{P}) = \mathbf{F}$ iff $v(A) \subseteq F_{\mathbb{P}}$ (this covers the case of impossibilities, when $v(A) = \emptyset$);
3. otherwise $V(A, \mathbb{P}) = \mathbf{N}$ (that is, $v(A) \cap T_{\mathbb{P}} \neq \emptyset$ and $v(A) \cap F_{\mathbb{P}} \neq \emptyset$).

4. The meaning of admissible answers

Recall, that besides direct answers to a question $?A$ we allow for other admissible answers to Q: these are expressions '$?A$ is not relevant' and 'Answer to $?A$ is not known'.

Consider a relevance model $\mathbb{N} = \langle \Omega, w, \mathbb{M} \rangle$, such that Ω is a topic which is not tautological (that is, $\Omega \neq U$). Suppose that a question of the form $?A$ is relevant w.r.t. $\mathbb{N}$. Thus one of the following holds:

1. $w(A) \subseteq \Omega$ and $w(A) = v(A)$, and, as $\Omega \neq U$, $v(\neg A) \not\subseteq \Omega$; or
2. $w(\neg A) \subseteq \Omega$ and $w(A) = v(A)$; thus $v(A)' \subseteq \Omega$, and, as $\Omega \neq U$, $v(A) \not\subseteq \Omega$.

As a result, at most one answer to a simple yes-no question is relevant w.r.t. a non-tautological topic. For a justification of this, see the next section.

Next, suppose that $?A$ is not relevant w.r.t. $\mathbb{N}$. Then neither $w(A)$ nor $w(\neg A)$ are subsets of Ω. This may cover two separate cases:

1. the value of $w(A)$ is determined, but neither $w(A)$ nor $w(\neg A)$ are subsets of Ω, or
2. the value of $w(A)$ is not determined, as w is just a partial function on $Form_{CPC}$.

There are further more fine-tuned distinctions possible if the weaker concept of relevance will be employed (see def. 6).

Finally, consider the statement ‘Answer to $?A$ is not known’. This in principle does not convey any information concerning relevance. One can argue that, in the case of *Mind Maze*, if $?A$ were relevant to a certain topic, enough information would be provided for an answer to $?A$ to be known, but in general this need not to be the case. We shall interpret this answer in terms of truth values as a claim that in a certain standard partition $\mathbb{P}$ being considered, the value of both answers to $?A$ is **N**. To formally represent such a claim we will be using the Łukasiewicz’s [14] operator I: $V(IA, \mathbb{P}) = \mathbf{T}$ iff $V(A, \mathbb{P}) = \mathbf{N}$.

One may notice that the fact that answer to $?A$ is not known may be interpreted also as the claim that information conveyed by answers to $?A$ is not needed in order to solve the task in question. As a result, another possible line of interpretation of this kind of answer is something like ‘epistemic irrelevance’, which can be expressed in terms of answers to $?A$ not being epistemic targets for an agent. However, we are not going to pursue this interpretation in the case of *Mind Maze*.

5. A toy example

Let us exemplify our ideas on a simple gameplay of a *20 questions* game. It should be noted that, however a toy example, it is not an artificial one, as there are some 7-years olds who consider this game rather useful in overcoming the ordeals of prolonged car travels. There are two parties in the game: an Answerer, who chooses an object, and a Questioner, who has 20 chances to guess what it is (there may be many Questioners involved). Only simple yes-no questions are allowed, but our admissible answers (‘not relevant’ and ‘not known’) are permitted as well.

Q1: Is it an animal?
A1: Yes.
Q2: Is it a mammal?
A2: Yes.
Q3: Is it a rodent?
A3: Yes.
Q4: Is it a rat?
A4: No.
Q5: Is it a pet?
A5: Yes.
Q6: Is it a guinea pig?
A6: No.
Q7: Is it a hamster?
A7: Yes.

Let us describe this gameplay in terms of sifting and funneling. The first question Q1 funnels the initial topic, which is the universe of objects. Then we have a sequence of funneling questions (Q2 funnels Q1, Q3 funnels Q2, etc.). An unsuccessful guess in Q4 may be interpreted as lack of relevance of ‘being a rat’ to the solution. It is then followed by another sequence of funneling questions (as Q5 funnels Q4 and Q6 funnels Q5). Finally, Q6 and Q7 are sifting questions w.r.t. the topic ‘A rodent which is a pet’. Notice, that also Q4, Q6 and Q7 may be interpreted as sifting questions, however w.r.t. a more general topic ‘A rodent’.

6. A case study

Now let us turn to slightly more complicated example. Our case study is based on a gameplay involving the story *The Traveller* (subject B4). The story goes as follows:

> A man, without a single visa, in one day visited eight different countries. Authorities of none of these countries tried to throw him out. What was his profession and how did he manage to do this?

The solution to the considered problem is that the man in question was a courier delivering post to the embassies. Thus there are two key pieces of information which the players needed to identify in order to reach the solution. The first one falls into the topic of the profession of the protagonist, the second one into the topic of visiting many countries in a day. Admittedly, the solution that the protagonist visited embassies and not the countries themselves draws somewhat on a popular belief that all the embassies are sovereign territories of the represented state. Although in fact most of them do not enjoy full extraterritorial status, this did not raise any issues in this research.

In modelling the subject’s solution to the task we use the following symbols to represent information involved (δ’s were put forward by the subject while ψ’s were gamemaster’s hints):

$T!$ – cognitive target of the subject (solution to the task);
Ω_1 – topic: profession;
Ω_2 – topic: visiting many countries in a day;
H_1 – the first key piece of information, such that $\nu(H_1) \subset \Omega_1$;
H_2 – the second key piece of information, such that $\nu(H_2) \subset \Omega_2$;
δ_1 – the protagonist’s activities were legal;
δ_2 – the protagonist travelled in a professional capacity;
δ_3 – the protagonist travelled during a global war;
δ_4 – the protagonist was an important public figure;
δ_5 – the protagonist visited 8 embassies;
δ_6 – the protagonist was an ambassador;
δ_7 – the protagonist was a caterer;
δ_8 – the protagonist was a security officer;
ψ_1 – the protagonist’s profession is useful for an embassy employees;
ψ_2 – the protagonist’s profession was a common one;
δ_9 – the protagonist was a cleaner;
δ_{10} – the protagonist’s professional duties were performed at the embassies;
δ_{11} – the protagonist was a postman.

The actual dialogue between the experimenter and the subject B4 can be found in [16]. A formal reconstruction of question processing in this gameplay runs as follows:

1. $T!$
2. $H_1 \wedge H_2 \rightarrow T$
3. $(H_1 \wedge H_2)!$
4. $(1)H_1, (1)H_2(v(H_1) \subset \Omega_1 \wedge v(H_2) \subset \Omega_2 \wedge (H_1 \wedge H_2 \rightarrow T))$
5. $v(\delta_1) \subset \Omega_2$
6. $?\delta_1$
7. δ_1
8. $v(\delta_2) \subset \Omega_1$
9. $v(\delta_2) \subset \Omega_2$
10. $?\delta_2$
11. δ_2
12. $v(\delta_3) \subset v(\delta_2)$
13. $?\delta_3$
14. $\neg\delta_3$
15. $v(\delta_3) \not\subset v(H_2)$
16. $v(\delta_4) \subset \Omega_1$
17. $v(\delta_4) \subset \Omega_2$
18. $?\delta_4$
19. $\neg\delta_4$
20. $v(\delta_5) \subset \Omega_2$
21. $?\delta_5$
22. δ_5
23. $v(\delta_5) = v(H_2)$
24. H_2
25. $v(\delta_6) \subset v(\delta_1) \cap v(\delta_2)$
26. $?\delta_6$
27. $\neg\delta_6$
28. $v(\delta_7) \subset (v(\delta_1) \cap v(\delta_2)) - v(\delta_6)$
29. $?\delta_7$
30. $\neg\delta_7$
31. $v(\delta_8) \subset (v(\delta_1) \cap v(\delta_2)) - (v(\delta_6) \cup v(\delta_7))$
32. $?\delta_8$
33. $\neg\delta_8$
34. $v(H_1) \subset v(\psi_1)$
35. $v(H_1) \subset v(\psi_2)$
36. $v(\delta_9) \subset (v(\delta_1) \cap v(\delta_2) \cap v(\psi_1) \cap v(\psi_2)) - (v(\delta_6) \cup v(\delta_7) \cup v(\delta_8))$
37. $?\delta_9$
38. $\neg\delta_9$
39. $v(\delta_{10}) \subset (v(\delta_1) \cap v(\delta_2) \cap v(\psi_1) \cap v(\psi_2)) - (v(\delta_6) \cup v(\delta_7) \cup v(\delta_8) \cup v(\delta_9))$
40. $?\delta_{10}$
41. $\neg\delta_{10}$
42. $v(\delta_{11}) \subset (v(\delta_1) \cap v(\delta_2) \cap v(\psi_1) \cap v(\psi_2)) - (v(\delta_6) \cup v(\delta_7) \cup v(\delta_8) \cup v(\delta_9) \cup v(\delta_{10}))$
43. $?\delta_{11}$
44. δ_{11}
45. $v(\delta_{11}) = v(H_1)$

46. H_1
47. T

Let us now focus on some elements of this reconstruction. The subject starts with setting the cognitive goal (lines 1st–4th) by identifying 'search areas' for two key pieces of information needed in order to solve the task. In other words, at this stage she identifies topics within which the elements of solutions are to be found, as expressed in the question in the 4th line. In the 5th line a topic of legality of the travel is identified as a subset of the topic Ω_2. On this basis the appropriate question is asked and an affirmative answer to it (6th line) confirms its relevance.

Now, let us turn to the lines 12th–15th. In the 12th line the topic of travelling during a global war is identified as a subtopic of travelling in a professional capacity. Negative answer to the appropriate question (lines 13th and 14th) leads to the conclusion that the 'global war' topic is not relevant w.r.t. the solution been sought and this line of inquiry is abandoned. Both 6th and 13th lines contain funneling questions: $?\delta_1$ funnels the topic Ω_2, whereas $?\delta_3$ funnels $?\delta_2$. The operation here is analogous to the one performed in the Q1 – Q4 sequence of our toy example: search areas within which solutions can be found are consecutively narrowed down.

In the lines 26th, 29th and 32nd we have three sifting questions w.r.t. the topic established as an intersection of $\nu(\delta_1)$ and $\nu(\delta_2)$ (that is, travelling legally in a professional capacity). The subject is trying to guess possible professions of the protagonist, thus she considers different topics at the very same level of generality; they are pairwise disjoint subsets of $\nu(\delta_1) \cap \nu(\delta_2)$. Another sequence of sifting questions is this: 36th, 39th, 42nd.

In the lines 23rd and 45th the key pieces of information are identified, which lead to the claim that the subject attained her cognitive target (line 47th).

7. Conclusion and further research

In this paper we outlined some formal tools for for analyses of question processing involved in solutions to a specific class of abductive problems. The relations of sifting and funneling, defined with respect to questions and topics, account well for our empirical data. Further studies shall focus on comparative analysis of the ways in which different subjects employ these relations in search for a solution. Also of interest are relationships which hold between sifting and funneling and other relations aimed at modeling questions dependency, in particular different versions of erotetic implication [8,1].

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